AD-753 486

THE NON-STATIONARY FLOW OF AN ANOMALOUS VISCOUS LIQUID

O. K. Fedorov

Foreign Technology Division
Wright-Patterson Air Force Base, Ohio

17 November 1972

AD753486

FTD-HT-23-1120-72

FOREIGN TECHNOLOGY DIVISION

THE NON-STATIONARY FLOW OF AN
ANOMALOUS VISCOUS LIQUID

by

O. K. Fedorov

Approved for public release;
distribution unlimited.

9

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
Foreign Technology Division
Air Force Systems Command
U. S. Air Force

2a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

2b. GROUP

3. REPORT TITLE
THE NON-STATIONARY FLOW OF AN ANOMALOUS VISCOUS LIQUID

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Translation

5. AUTHOR(S) (First name, middle initial, last name)
Fedorov, O.K.

6. REPORT DATE
1970

7a. TOTAL NO. OF PAGES
9

7b. NO. OF REFS
2

8a. CONTRACT OR GRANT NO.

b. PROJECT NO. 7343

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
FTD-HT-23-1120-72

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Foreign Technology Division
Wright-Patterson AFB, Ohio

13. ABSTRACT

The author solves the problem of a non stationary flow in half space of liquid (the problem of Rayleigh), having the following property: ~~at~~ displacement speed ~~gamma less~~ than gamma$_0$ than ~~gamma$_0$ with a~~ viscosity ~~of~~ Mu$_1$ but at gamma greater than gamma$_0$ it has a viscosity of Mu$_2$. It is assumed that the liquid was initially at rest. At the initial moment of time the surface, on top of which the liquid is found, begins to move with constant speed. Thus a boundary arises separating liquids with viscosities Mu$_1$, and Mu$_2$. On this boundary forward speed and drift speed are to be continuous. The solution is expressed through a Laplace function. In the statement of the problem considered by the author surface forces should act on the separation surfaces, because each stress in the passage from one region to another undergoes an interruption.
AP2005114

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Mathematic Method						
Fluid Flow						
Fluid Viscosity						

FTD-HT- 23-1120-72

EDITED TRANSLATION

FTD-HT-23-1120-72

THE NON-STATIONARY FLOW OF AN ANOMALOUS VISCOUS LIQUID

By: O. K. Fedorov

English pages: 4

Source: Trudy. Leningradskogo Tekhnologicheskogo Instituta Tsellyulozno-Bumazhnoy Promyshlennosti, Izd-vo Lesnaya Promyshlennost', Moscow, No. 25, 1970, pp 22?-224.

Requester: ASD

Translated: Charles T. Ostertag Jr.

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

THE NON-STATIONARY FLOW OF AN ANOMALOUS VISCOUS LIQUID

O. K. Fedorov

During the flow of a dispersed system the effect of a sharp drop in viscosity is observed at a certain rate of deformation of displacement [1]. Such an anomaly in viscosity can lead to a unique situation when two areas of liquid flow are formed: an area of low velocities of displacement, characterized by a high value of viscosity, and a low-viscosity area, corresponding to high velocities of displacement. Below an anlaysis is made of the problem of the plane-parallel flow of such a liquid caused by the movement of flat wall.

During the flow of an anomalous viscous liquid the stress of the displacement τ is connected with the velocity of displacement $\dot{\gamma}$ by the dependence

$$\tau=\mu(\dot{\gamma})\dot{\gamma}, \qquad (1)$$

where $\mu(\dot{\gamma})$ – viscosity, depending on the velocity of displacement.

At $\dot{\gamma} = \dot{\gamma}_0$ there is a drop in viscosity, since the flow of a liquid at $\dot{\gamma} \leq \dot{\gamma}_0$ can be characterized by the viscosity μ_1 (the greatest Newtonian viscosity), and at $\dot{\gamma} > \dot{\gamma}_0$ – by the viscosity μ_2 (the least Newtonian viscosity).

The liquid occupies the half space $y > 0$ and is in contact with the wall $y = 0$. The wall moves uniformly at a constant rate u directed along the axis X. The velocity v of plane-parallel flow of a liquid will be a function of time t and coordinate y. The field of velocities of displacement deformation of the liquid corresponds to two values of viscosity μ_1 and μ_2, characterizing the two areas of flow. The surface of separation between the areas is determined by coordinate Y(t), whereas on this surface the conditions of continuity of the field of velocities $v_{1,2}$ and the field of velocities of displacement $\dot{\gamma}_{1,2}$ should be satisfied (subscripts 1, 2 are taken for areas with the corresponding values of viscosity μ_1 and μ_2). On the moving wall $\dot{\gamma} = 0$, therefore the area with viscosity μ_1 adjoins it.

It follows from the Navier-Stokes equations that without a calculation for the pressure gradient

$$\frac{\partial v_i}{\partial t} = a_i \frac{\partial^2 v_i}{\partial y^2} (i = 1, 2), \tag{2}$$

where $v_1 = \frac{\mu_1}{\rho}$, ρ – density of the liquid.

Let us present the boundary conditions in the form

$$v_1 = u \text{ when } y = 0, \; v_2 = 0 \text{ when } y \to \infty, \tag{3}$$

$$v_1 = v_2, \; \frac{\partial v_1}{\partial y} = \frac{\partial v_2}{\partial y}, \text{ when } y = Y(t). \tag{4}$$

Serving for the determination of the surface of separation is the condition that the velocity of displacement on it is equal to $\dot{\gamma}_0$:

$$\frac{\partial v_1}{\partial y} = -\dot{\gamma}_0 \text{ at } y = Y(t). \tag{5}$$

In such a formulation the examined problem is close to the known Stefan problem concerning the freezing of ice [2].

The solution of equation (2), satisfying the boundary conditions (3), we present in the form

$$v_1 = u - A\Phi\left(\frac{y}{2\sqrt{a_1 t}}\right)$$

and

$$v_2 = B\left[1 - \Phi\left(\frac{y}{2\sqrt{a_2 t}}\right)\right].$$

Here

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi$$

is the Laplace function; A and B - arbitrary constants. Using conditions (4) and (5) we obtain

$$v_1 = u - \frac{Y\dot{\gamma}_0}{\lambda_0 \alpha}\frac{\Phi(\alpha\lambda)}{\Phi'(\alpha\lambda_0)} \quad (6)$$

and

$$v_2 = \frac{Y\dot{\gamma}_0}{\lambda_0}\frac{1 - \Phi(\lambda)}{\Phi'(\lambda_0)}, \quad (7)$$

where

$$\lambda = \frac{y}{2\sqrt{a_2 t}}, \quad \lambda_0 = \frac{Y}{2\sqrt{a_2 t}}, \quad \alpha = \sqrt{\frac{a_2}{a_1}},$$

$$\Phi'(x) = \frac{d\Phi(x)}{dx},$$

whereas Y(t) is determined from the equation

$$\frac{1 - \Phi(\lambda_0)}{\Phi'(\lambda_0)} + \frac{1}{\alpha}\frac{\Phi(\alpha\lambda_0)}{\Phi'(\alpha\lambda_0)} = \frac{u\lambda_0}{Y\dot{\gamma}_0}. \quad (8)$$

Expressions (6) and (7) are the solutions of the problem which satisfy the following conditions: f = 0, Y = 0, v_2 = 0, i.e., at the initial moment all the liquid is stationary.

We will make use of the fact that usually $\mu_1 >> \mu_2$; then $\alpha << 1$. Using for the Lapalce function the concept in the form of a series and retaining in it only the first member, we obtain the following expression for the velocity of flow of a liquid in a highly viscous area

$$v_1 = u - \dot{\gamma}_0 y. \qquad (9)$$

We arrive at such an expression if we examine the flow of a liquid at $f \to \infty$. From here it follows that in a highly viscous wall zone the flow is established very rapidly and it depends on the rate of movement of the wall u and the velocity of displacement $\dot{\gamma}_0$.

BIBLIOGRAPHY

1. Ребиндер П. А. Дополнения к переводу реологии. Реология, ИЛ, 1962.
2. Карслоу Г., Егер Д. Теплопроводность твердых тел. Изд-во «Наука», 1964.